On the physical meaning of the Unruh effect

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Abstract

We present simple arguments that detectors moving with constant acceleration (even acceleration for a finite time) should detect particles. The effect is seen to be universal. Moreover, detectors undergoing linear acceleration and uniform, circular motion both detect particles for the same physical reason. We show that if one uses a circularly orbiting electron in a constant external magnetic field as the Unruh–DeWitt detector, then the Unruh effect physically coincides with the experimentally verified Sokolov–Ternov effect.

Hawking radiation [1] and the closely related Unruh [2] radiation are often seen as first steps toward combining general relativity and quantum mechanics. Under achievable conditions for gravitational system these effects are too small to be experimentally testable. In this letter we examine the physical meaning of the Unruh effect and argue that for uniform, circular acceleration the Unruh effect has already been observed. Given the close connection between the Hawking and Unruh effects this experimental evidence for the latter gives strong support for the former.

It has been shown [2] that a detector moving eternally with constant, linear acceleration a should detect particles with Planckian distribution of temperature $T = a/2\pi$. The non–inertial reference frame which is co–moving with the detector has an event horizon. Even massless particles radiated a distance $1/a$ behind the detector would never catch up with an eternally accelerating detector. It is the reference frame co–moving with the *eternally* accelerating detector which "sees" the Rindler metric. Thus it seems that the Unruh effect is strongly related to the existence of the horizon. However, if the effect only exists for an eternally accelerating observer/detector then it can be discarded as unphysical since one can never have a detector that undergoes constant acceleration from infinite past time to infinite future time. Due to the Hawking radiation [1] black-holes do not exist eternally. As well a positive cosmological constant (giving a de-Sitter space–time) should eventually be radiated away to zero.

The real question is whether or not a detector which moves with linear, constant acceleration for a finite time will see particles (e.g. a detector which is initially stationary, accelerates for a finite time and then continues with constant velocity). We are interested whether the detector gets excited or not during the period when it moves homogeneously. We are not interested

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in the detector's reaction during the periods when the acceleration is turned on or off. The reaction of the detector which we are interested in does not come from internal forces where one part of the detector can move with respect to another (like the arrow of an ammeter which moves with respect to its box if it is shaken), but is due to the existence of a universal radiation in the detector's non–inertial reference frame. We consider two kinds of homogeneous accelerations: (i) from a force that is constant in magnitude and direction resulting in linear accelerated motion; (ii) from a force that is constant only in magnitude resulting in circular motion. We take as our definition of a particle that thing which causes a detector to click, i.e. jump from one of its internal energy levels to a higher one. We do not know any other *invariant* definition of a particle.

If detectors do click during homogeneous, accelerated motion occurring for a finite time, then the Unruh effect does not depend on the existence of a horizon[3], since for finite time acceleration the co–moving frame "sees" a metric different from Rindler and does not have a horizon: a massless particle with light speed velocity following the detector will eventually catch up with it if the detector accelerates for finite time.

Once this idea is accepted, we can go further and state that there is no significant physical difference between detectors in homogeneous, linear acceleration versus uniform circular motion. Note, the reference frame co–moving with the detector performing eternal homogeneous, circular motion does not have a horizon (only a light–surface). A particle can eventually catch up with a circularly moving detector. Previous investigations on whether or not a moving detector clicks or not under various assumptions about its motion can be found in references [3] [4]

In this letter we show (following other authors) that detectors performing homogenous linear and circular accelerations (or any other homogeneous non–inertial motion in the empty Minkowski space) do detect particles, and they do this for the same physical reason. Moreover, we show that the circular Unruh effect has been well known for a long time under a different name and has even been experimentally observed.

In all cases we consider Minkowski space–time, and take $\hbar = 1$ and $c = 1$. For simplicity we consider a linear interaction of the detector with a *free* scalar field. We consider the following two processes: (i) the detector is originally in its ground state and then gets excited because of its non–inertial motion; (ii) the detector is originally in its excited state and then relaxes to its ground state. In both cases the background QFT is originally in its ground state. We want to find the probability rates for these two processes. As a result of these processes the background QFT will become excited, i.e. the detector will radiate quanta of the background QFT when performing the above two processes.

To leading order in perturbation theory the probability rate per unit time is [5]:

$$w_{\mp} \propto \int_{-\infty}^{+\infty} d\tau \, e^{\mp \mathrm{i}\, \Delta\mathcal{E}\, \tau} \, G\left[x(t-\tau/2) \ , \ x(t+\tau/2)\right], \tag{1}$$

where t is the detector's proper time; $\Delta\mathcal{E} = \mathcal{E}_{up} - \mathcal{E}_{down} > 0$ is the discrete change of the detector's internal energy level; the "$-$" sign, both in the LHS and in the exponent, corresponds to the first process, while the "$+$" sign corresponds to the second process mentioned above; $G\left[x(t-\tau/2),\, x(t+\tau/2)\right] = \left\langle 0 \left| \phi[x(t-\tau/2)] \ \ \phi[x(t+\tau/2)] \right| 0 \right\rangle$ is the Wightman function of

[3]Here we understand notion of the horizon as the *eternally* existing surface from inside of which classically nothing can *ever* escape.

the scalar field ϕ. This function measures the correlation between fluctuations of the scalar field at two points in the space–time in the vacuum of the scalar QFT. In our case these two points are on the same trajectory $x(t)$. Because of this these points are causally connected to each other even for the eternally, linearly accelerating detector. However, as we will see below the important contribution to $w_\mp$ in all cases comes from the imaginary τ.

The reason why we consider the detector approach to the Unruh effect is that then all our considerations can be made completely generally covariant [6]. This allows us to address the question as to whether or not a detector making a particular motion in Minkowski space–time sees/detects particles.

Eq. (1) shows that the probability rates $w_\mp$ are Fourier images of the Wightman function. The Wightman function is a universal characteristic of the field, and its features *universally* characterize the reaction of a detector moving along the trajectory $x(t)$. Of course the spectrum of the detected particles depends on the detector's trajectory.

Note that eq.(1) is written for the simplest linear type of interaction of the detector with ϕ [5] [6]. In cases with a more complicated interaction, say non–linear or via derivatives of the field, one would get probability rates that are Fourier images of powers or derivatives of the Wightman function. It will be clear from the discussion below that this would not change the spectrum of the detected particles, but would only alter the time necessary to reach the equilibrium distribution over the detector's energy levels under the homogeneous background radiation.

Thus, the question is reduced to the study of the characteristic features of the Wightman function of free massless particles:

$$G(x,y) = \frac{1}{|x_0 - y_0 - \mathrm{i}\,\epsilon|^2 - |\vec{x} - \vec{y}|^2}, \tag{2}$$

with various homogeneous trajectories – $x(t_1) = x$ and $x(t_2) = y$ – plugged into it. Below we are going to consider three different trajectories. All poles of the two–point correlation functions (both in coordinate and momentum spaces) have physical meanings based on intuition from condensed matter physics.

In the case of motion with constant velocity one can show that (see e.g. [6]): $w_- = 0$, and $w_+ \propto \Delta\mathcal{E}$. The physical meaning of this result is as follows: If the detector moves with constant velocity in the vacuum of a QFT there is zero probability for it to get excited, $w_- = 0$. However, if the detector was originally in the excited state, there is a non–zero probability for it to radiate spontaneously, $w_+ \neq 0$.

For the case of *eternal*, constant, linear acceleration – $x(t) = \left(\frac{1}{a}\sinh\left[a\,t\right],\, \frac{1}{a}\cosh\left[a\,t\right],\, 0,\, 0\right)$ with t the detector's proper time and a its acceleration – the Wightman function is:

$$G\left[x(t-\tau/2)\ \ ,\ \ x(t+\tau/2)\right] \propto \frac{a^2}{\sinh^2\left[\frac{a}{2}\,\left(\tau - \mathrm{i}\,\epsilon\right)\right]}. \tag{3}$$

The integral in eq.(1) is taken using contour integration in the complex τ plane. Since $\Delta\mathcal{E} > 0$, the integral w_- in eq.(1) uses a contour which is closed with a large, clockwise semi-circle in the lower complex half–plane. This contour is denoted by C_-. For w_+ the contour is closed with a large, counterclockwise semi-circle in the upper complex half–plane, and is denoted by C_+. This choice of contours for $w_\mp$ is used everywhere below.

Unlike the constant velocity case, the Wightman function now has non–trivial poles encircled by the C_- contour, hence, $w_- \neq 0$. The positions of the poles are easy to find, so the integral in eq.(1) can be calculated exactly with the result:

$$w_- \propto \frac{\Delta\mathcal{E}}{e^{\frac{2\pi\Delta\mathcal{E}}{a}} - 1}, \quad w_+ \propto \Delta\mathcal{E}\left[1 - \frac{1}{e^{\frac{2\pi\Delta\mathcal{E}}{a}} - 1}\right]. \tag{4}$$

Therefore a detector moving with constant acceleration in the vacuum of the background QFT does detect particles. The detected particles have a Planckian distribution with temperature $T = \frac{a}{2\pi}$ [2]. The detector gets excited because there is a non–trivial correlation between field excitations of ϕ along its trajectory. The nontrivial contribution to w_- comes from the non–trivial poles in the complex τ plane at $\tau = 2\pi\,\mathrm{i}\,n/a$, where n is negative integer number. Note that along the trajectory of a detector fixed at a spatial point in the vicinity of a Schwarzschild black hole the Wightman function will have the same analytic features, i.e. the detector will click for the same physical reason as the accelerating one.

Is it really physically correct to take into account the contributions of such poles? They are definitely present for eternal, linear acceleration. However, if one considers a more realistic linear acceleration with starting/stopping of the accelerations these initial/final conditions increase the difficulty of the analysis making it much harder to get a clear physical picture of what is going on.

Instead of performing a new calculation for a finite time, linearly, accelerating detector we turn our attention to circular motion. We will consider homogeneous circular motion, i.e. eternal circular motion with no starting or stopping. We argue – via the specific example where our two–energy level detector is a electron in an external magnetic field – that homogeneous circular motion is a good approximation for real circular motion with a starting/stopping times. Moreover, in this type of detector the contribution of the non–trivial poles has been *experimentally verified.*

Now, following [7], we show that non–trivial poles appear in the case of a homogeneously orbiting detector interacting with ϕ. The trajectory of such a detector with radius R and angular velocity ω_0, is $x(t) = (\gamma\, t,\, R\cos[\gamma\,\omega_0\, t],\, R\sin[\gamma\,\omega_0\, t],\, 0)$, $\gamma = 1/\sqrt{1 - R^2\,\omega_0^2}$ and t is the detector's proper time.

Inserting this trajectory into eq.(2), we obtain:

$$G\Big[x(t-\tau/2)\ ,\ x(t+\tau/2)\Big] \propto \frac{1}{\left[\gamma\ (\tau - \mathrm{i}\,\epsilon)\right]^2 - 4\,R^2\,\sin^2\left[\frac{\gamma\,\omega_0}{2}\,\tau\right]}. \tag{5}$$

This two–point correlation function has poles in the lower complex τ plane enclosed by C_-. These poles are similar in nature to those of the Wightman function for a heat bath [6] or for linear acceleration eq.(3), which lead to a Boltzmann type exponential contribution to $w_\mp$.

For the case of circular motion the velocity is $v = \omega_0\, R\,\gamma$ and the acceleration is $a = \gamma^2\,\omega_0^2\, R$ in the instantaneously, co–moving inertial frame. Unlike the case of eternal, linear acceleration the integral in eq.(1) for $w_\mp$ for the case of orbiting motion can not be done exactly, since we do not know the exact position of all the poles in eq.(5). However, assuming that the energy splitting is not too small (i.e. $\Delta\mathcal{E} > a$) we can approximately find the probability rate [7]:

$$w_- \propto a\, e^{-\sqrt{12}\,\frac{\Delta\mathcal{E}}{a}}, \quad w_+ \propto a\,\left(e^{-\sqrt{12}\,\frac{\Delta\mathcal{E}}{a}} + 4\sqrt{3}\,\frac{\Delta\mathcal{E}}{a}\right). \tag{6}$$

The exponential contributions come from the non–trivial poles in (5) at $\tau \approx \pm \mathrm{i}\sqrt{12}/a$. The non–exponential contribution to w_+ comes from the trivial pole at $\tau = \mathrm{i}\,\epsilon$, and is present even if $a = 0$, i.e. corresponds to spontaneous radiation.

Whereas eq.(4) implies a thermal spectrum for linear acceleration, the results of eq.(6) show that the spectrum observed by an orbiting detector is not thermal. Intuition from condensed matter informs us that the Planckian distribution is strongly related to the form of the two–point correlation function in eq.(3). The two–point function for circular motion, given in eq.(5), has a drastically different form from that in eq.(3).

Thus, we see that the circular Unruh effect has the same physical origin as the linear case: detectors in homogeneous motion get excited due to non–trivial correlations between field fluctuations along their trajectories. Now we are going to show that the circular Unruh effect has been well known for a long time but under the name "Sokolov–Ternov effect". Since the Sokolov–Ternov effect is experimentally verified this shows that the non–trivial poles are not simply a mathematical abstraction, but have a physical meaning.

Interestingly the same Wightman function just investigated for the orbiting observer appears in the calculation of the Sokolov–Ternov effect [8]. This is not a coincidence. See in particular the derivation of the Sokolov–Ternov effect in [6] [9]. We repeat the main steps of this calculation, but for an arbitrary gyromagnetic number g.

The Sokolov–Ternov effect describes the partial depolarization of electrons in a magnetic field in storage rings due to synchrotron radiation. It is well known that electrons in circular motion radiate due to their charge. Apart from this electrons have two energy levels in an external constant magnetic field bending their trajectories: with their spins along or against the direction of the magnetic field. Hence, they can also radiate via flips of their spins. This spin flip radiation is strongly suppressed in comparison with the radiation due to the electric charge [10].

At first it seems that the spin flip radiation should eventually polarize the electron beam completely. However, the flips can happen in both "directions" — either decreasing or increasing the spin energy. Due to the latter effect the polarization is not complete. To understand the relation of this effect to the Unruh effect let us, first, note that electrons can be considered as quasi-classical detectors (such as the Unruh–DeWitt detector with two energy levels) when they move ultra–relativistically. In this case we can neglect both quantization of their motion and back-reaction to the photon radiation. Apart from this in the non–inertial, co–moving reference frame the electrons are at rest. The spin flip transition which decreases the spin energy can happen due to spontaneous radiation. But what is the reason for the spin flip transition which increases the spin energy in this frame where the electrons are at rest? We will show that the latter transition happens due to existence of the universal radiation in the non–inertial co–moving reference frame, i.e. for the same physical reason as in the case of the Unruh effect appearing for the detector interacting with ϕ. Posed another way — the effect appears due to the non–trivial field correlations along the orbiting trajectory of the electrons.

The probability rate of synchrotron radiation from a spin flip, can be obtained from the relativistic equation of motion for a spin $\vec{s}$ as given by [9]:

$$\begin{aligned} \frac{d\vec{s}}{dt} &= \mathrm{i}\,\left[\hat{H}_{int},\,\vec{s}\right], \\ \hat{H}_{int} &= -\frac{e}{m}\,\vec{s}\,\left[\left(\alpha+\frac{1}{\gamma}\right)\,\vec{H} - \frac{\alpha\,\gamma}{\gamma+1}\,\vec{v}\,\left(\vec{v}\cdot\vec{H}\right) - \left(\alpha+\frac{1}{\gamma+1}\right)\,\vec{v}\times\vec{E}\right], \end{aligned} \tag{7}$$

where t is now the laboratory time, $\alpha = (g-2)/2$, $\vec{v}$ is the particle's velocity, $\gamma = 1/\sqrt{1-v^2}$ and $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields. Using the interaction Hamiltonian from eq.(7) we can derive the probability rates for photon emission with spin flips [6]:

$$w_\mp \propto \oint_{C_\mp} d\tau\, e^{\mp \mathrm{i}\,\omega_s\,\tau}\, \hat{W}\; \left.\frac{1}{\left(\tau - \mathrm{i}\,\epsilon\right)^2 - \left(\vec{r} - \vec{r}'\right)^2}\right|_{r=r\left(t-\frac{\tau}{2}\right),\; r'=r\left(t+\frac{\tau}{2}\right)} . \tag{8}$$

$\hat{W}$ [6] is a differential operator acting on t and r. It appears due to the fact that our "detectors" interact with the electric and magnetic fields rather than directly with the vector–potential (see eq.(7)).

Now in eq.(8) we insert for $r(t)$ a homogeneous circular trajectory: $(t, R\,\cos\omega_0 t, R\,\sin\omega_0 t, 0)$ with laboratory time, t. We can do this, despite the fact that the real motion of electrons has starting/stopping points, because the main contribution to the integral in eq.(8) comes from very small times τ (to understand this point one should examine the alternative stationary phase calculations of the probability rates $w_\mp$ in [8] and [10]). Thus, in eq.(8) we have the same Wightman function as in eq.(5). Note that $\Delta\mathcal{E}$ is replaced by $\omega_s = [1 + \gamma\,(g-2)/2]\,\omega_0$ and $\omega_0 = e\,H_b/\mathcal{E}$ is the cyclotron frequency of an electron with energy, $\mathcal{E}$, in constant background magnetic field, H_b; ω_s is the energy difference between electron's spin states in a constant, background magnetic field. The differential operator $\hat{W}$ is the source of the difference between the standard Sokolov–Ternov and circular Unruh effects for detectors interacting with scalar fields.

Taking the integral in eq.(8), and considering only $\alpha > 0$ yields [6] [10] :

$$w_\mp \approx \frac{5\sqrt{3}\,e^2\gamma^5}{16\,m^2\,R^3}\,\left\{F_1(\alpha)\,e^{-\sqrt{12}\,\alpha} + F_2(\alpha) \mp F_2(\alpha)\right\}, \tag{9}$$

where

$$\begin{aligned} F_1(\alpha) &= \left(1 + \frac{41}{45}\,\alpha - \frac{23}{18}\,\alpha^2 - \frac{8}{15}\,\alpha^3 + \frac{14}{15}\,\alpha^4\right) - \frac{8}{5\sqrt{3}}\,\left(1 + \frac{11}{12}\,\alpha - \frac{17}{12}\,\alpha^2 - \frac{13}{24}\,\alpha^3 + \alpha^4\right), \\ F_2(\alpha) &= \frac{8}{5\sqrt{3}}\,\left(1 + \frac{14}{3}\,\alpha + 8\,\alpha^2 + \frac{23}{3}\,\alpha^3 + \frac{10}{3}\,\alpha^4 + \frac{2}{3}\,\alpha^5\right). \end{aligned} \tag{10}$$

Note the exponential factor in eq.(9), which appears for the same reason as the one in eq.(6): in both cases the Wightman functions have the same pole in the lower complex τ plane. If $g = 2$ (i.e. $\alpha = 0$) we obtain the standard Sokolov–Ternov expression:

$$w_\mp \approx \frac{5\sqrt{3}}{8}\,\frac{e^2\,\gamma^5}{m^2\,R^3}\,\left(1 \mp \frac{8\sqrt{3}}{15}\right). \tag{11}$$

In this case the exponent is equal to 1. This is the reason why the exponential factor, the hallmark of the Unruh effect, is usually overlooked in the standard Sokolov–Ternov considerations. Note that the exponential factor is always present in the form $e^{1/\gamma}$ even if g is exactly 2, but we are taking $\gamma \gg 1$. In any case, if we consider $g \neq 2$, then the exponent is explicitly present. In the case of the Sokolov–Ternov effect we have $\Delta\mathcal{E}/a \approx (g-2)/2$ if $\gamma \gg 1$. Thus, the laboratory observer interprets the effect as the Sokolov–Ternov effect, while the non-inertial co-moving observer interprets the effect as the circular Unruh effect. Physically these two effects are the

same. The connection between the Unruh and Sokolov-Ternov effects has been previously discussed in [7], [11] and [12].

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